Practice Problems

1. A fair die is cast until a 6 appears. What is the probability that it must be cast more than 5 times?

2. Prove that for any three events A, B and C, each having positive probability,

$$P(A \cap B \cap C) = P(A)P(B \mid A)P(C \mid A \cap B)$$

Hint: For any two events with positive probability we have $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

3. If the probability of hitting a target is $\frac{1}{5}$, and ten shots are fired independently, what is the probability of the target being hit at least twice? What is the conditional probability that the target is hit at least twice, given that it is hit at least once?

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4. Generalized linear models (GLMs) form a class of statistical models for response variables whose distribution belongs to the exponential dispersion family. This family is characterized by the following common form for the pdf/pmf

$$f_Y(y;\theta,\phi) = \exp\left\{\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right\}$$

(a) Determine whether or not the Normal Distribution $Y \sim N(\mu, \sigma^2)$ belongs to the exponential dispersion family, where
$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{1}{2\sigma^2}(y-\mu)^2)$
If it is a member of the family, give expressions for
$\theta, \phi, a(\phi), b(\theta), \text{and} c(y,\phi)$.

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4. (b) Additionally if Y belongs to the exponential dispersion family, use the following facts to write down the mean and variance:

$\text{E[Y]} = \frac{\partial}{\partial\theta} b(\theta)$ and $\text{Var[Y]} = \frac{\partial^2}{\partial\theta^2} b(\theta) a(\phi)$

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5. Similarly, show that the inverse Gaussian distribution belongs to the exponential dispersion family.

$$f(y|\eta,\lambda) = \sqrt{\frac{\lambda}{2\pi y^3}} \exp\left\{-\frac{\lambda}{2}\left(\eta^2 y + y^{-1} - 2\eta\right)\right\}$$

Write the canonical parameter, θ, and dispersion parameter, ϕ, as a function of λ and η.